

ABSOLYUT VA NISBIY XATOLIKLARNI ANIQLASH.*Burhonova Mohichehra Esonboy qizi**Mirzo Ulug'bek nomidagi O'zMU Jizzax filiali talabasi*

Annotatsiya. Ushbu maqolada dastlab xatolik tushunchasiga ta'rif beramiz. Absolyut va nisbiy xatoliklar bo'yicha tushuncha va ma'lumotlarga ega bo'lamiz. Aniq va nisbiy xatoliklar o'rtasidagi farqni, ularni yechish usullarini ko'rib chiqamiz. Ikki sonning arifmetik amallarining absolyut va nisbiy xatoliklarini topish formulalarini o'rganamiz.

Kalit so'zlar: xatolik, absolyut, nisbiy, limit, yig'indi, ayirma, taqribiy, daraja.

Ixtiyoriy matematik masalani sonli yechishda biz aniq yechimga ega bo'lmadan, balki yechimni u yoki bu darajadagi aniqlikda topamiz. Aniq yechim bilan taqribiy yechim orasidagi farq *xato* deyiladi.

Agar A - biror miqdorning aniq qiymati bo'lib, a uning ma'lum taqribiy qiymati bo'lsa, u vaqtda a sonning *absolut xatoligi* deb $\Delta a = |A - a|$ ga aytiladi.

Absolut xatodan kichik bo'lmagan har qanday songa taqribiy a sonning *limit absolut xatosi* $\Delta(a)$ deb aytiladi. Bu ta'rifdan $|A - a| \leq \Delta(a)$, bundan esa $a - \Delta(a) \leq A \leq a + \Delta(a)$ kelib chiqadi.

Absolut xato va limit absolut xato hisoblash xatoligini baholash uchun yetarli emas. Misol uchun, ikkita og'irlik o'lchanganda $m_1 = 110,2 \pm 0.2 g$ va $m_2 = 11,2 \pm 0.2 g$ natijalar hosil bo'lsin, bu yerda har ikkalasida limit absolut xatolik bir xil bo'lishidan qat'i nazar birinchi o'lchash ikkinchi o'lchashdan ancha aniqdir. Aniqlikni yaxshiroq baholaydigan tushuncha kiritamiz.

Absolut xatoning taqribiy sonning absolut qiymatiga nisbati taqribiy sonning *nisbiy xatosi* δ_a deb aytiladi:

$$\delta_a = \frac{\Delta a}{|a|}$$

Xuddi yuqoridagidek *limit nisbiy xato* $\delta_{(a)}$ tushunchasi kiritiladi:

$$\delta_{(a)} = \frac{\Delta(a)}{|a|}$$

Limit nisbiy xatolik yordamida A son quyidagicha yoziladi:

$$A = a(1 \pm \delta_{(a)})$$

Taqribiy sonlar ustida amallar natijalarining xatoligi:

1. Ikki son yig'indisi yoki ayirmasining absolut xatoliklari:

Sonlarni qo'shish va ayirishda ularning absolut xatoliklari qo'shiladi:

$$\Delta(a \pm b) \leq \Delta(a) + \Delta(b) \leq \Delta_a + \Delta_b = \Delta_{a \pm b}$$

2. Ikki son yig'indisi va ayirmasining nisbiy xatoliklari:

$$\begin{aligned} \delta_{(a+b)} &= \frac{\Delta(a+b)}{|a+b|} = \frac{\Delta(a) + \Delta(b)}{|a+b|} = \frac{|a| * \delta_{(a)} + |b| * \delta_{(b)}}{|a+b|} \\ &\leq \frac{|a| * \delta_{max} + |b| * \delta_{max}}{|a+b|} = \delta_{max} \end{aligned}$$

$$\delta_{(a-b)} = \frac{\Delta(a-b)}{|a-b|} = \frac{\Delta(a) + \Delta(b)}{|a-b|} \leq \nu \delta_{max}$$

3. Ikki son bir-biriga ko'paytirilganda va bo'lingandagi absolut xatoliklari:

$$\Delta(a * b) = a\Delta(b) + b\Delta(a) \leq a\Delta_b + b\Delta_a$$

$$\Delta\left(\frac{a}{b}\right) = \frac{b\Delta(a) + a\Delta(b)}{b^2} \leq \frac{a\Delta_b + b\Delta_a}{b^2}$$

4. Ikki son bir-biriga ko'paytirilganda va bo'lingandagi nisbiy xatoliklari:

$$\delta_{(ab)} \leq \delta_{(a)} + \delta_{(b)}$$

$$\delta\left(\frac{a}{b}\right) \leq \delta_{(a)} + \delta_{(b)}$$

5. Taqribiy son darajaga ko'tarilganda uning nisbiy xatoligi daraja ko'rsatgichiga ko'paytiriladi:

$$\delta_{(a^k)} \leq k\delta_{(a)} \quad \text{va} \quad \delta_{(\sqrt[k]{a})} \leq \frac{\delta_{(a)}}{k}$$

Quyidagi funksiyalarning absolyut va nisbiy xatoligini aniqlang.

Misol-1.

$$y = \frac{ab}{\sqrt[3]{c}}, \quad a = 3.85 \pm 0.02; \quad b = 2.0435 \pm 0.003; \quad c = 962.6 \pm 0.2$$

1-usul.

$$\delta_a = \frac{\Delta_a}{|a|} = \frac{0.02}{3.85} = 0.005194805194805 \approx 0.005195$$

$$\delta_b = \frac{\Delta_b}{|b|} = \frac{0.003}{|2.0435|} = 0.001468069488622 \approx 0.001468$$

$$\delta_c = \frac{\Delta_c}{|c|} = \frac{0.2}{|962.6|} = 0.000207770621234 \approx 0.000208$$

$$\delta_y = \delta_a + \delta_b + \frac{1}{3} * \delta_c = 0.005195 + 0.001468 + \frac{1}{3} * 0.000208 = 0.006732(3) \\ \approx 0.006732$$

Bu degani nisbiy xatolik $\delta_y = 0.006732 * 100\% = 0.6732\%$.

$$y = \frac{ab}{\sqrt[3]{c}} = \frac{3.85 * 2.0435}{\sqrt[3]{962.6}} = 0.7968075108 \approx 0.796808$$

$$\Delta_y = y * \delta_y = 0.796808 * 0.006732 = 0.005364111456 \approx 0.005364$$

Javob: Absolyut xatolik $\Delta_y = 0.005364$ va nisbiy xatolik $\delta_y = 0.6732\%$.

2-usul.

$$y_{max} = \frac{3.87 * 2.0465}{\sqrt[3]{962.4}} = 0.8021781762464 \approx 0.802178$$

$$y_{min} = \frac{3.83 * 2.0405}{\sqrt[3]{962.8}} = 0.7914497493244 \approx 0.791500$$

$$y_{ABS} = \frac{y_{max} - y_{min}}{2} = \frac{0.802178 - 0.791500}{2} = \frac{0.010678}{2} = 0.005339.$$

$$y_{nis} = \frac{y_{ABS}}{\frac{y_{max} + y_{min}}{2}} = \frac{0.005339}{\frac{0.802178 + 0.791500}{2}} = \frac{0.005339}{0.796839} = 0.006700224261 \\ \approx 0.00670.$$

Bu degani nisbiy xatolik $y_{nis} = 0.00670 * 100\% = 0.67\%$

Javob: Absolyut xatolik $y_{ABS} = 0.005339$ va nisbiy xatolik $y_{nis} = 0.67\%$

Misol-2.

$$y = \frac{(a + b) * m}{\sqrt{c - d}}, \quad a = 23.16 \pm 0.02; \quad b = 8.23 \pm 0.005; \quad c = 145 \pm 0.08; \quad d \\ = 28.6 \pm 1; \quad m = 0.28 \pm 0.006$$

1-usul.

$$\delta_{a+b} = \frac{\Delta_a + \Delta_b}{|a + b|} = \frac{0.02 + 0.005}{|23.16 + 8.23|} = \frac{0.025}{31.39} = 0.0007964319 \approx 0.000796$$

$$\delta_m = \frac{\Delta_m}{|m|} = \frac{0.006}{|0.28|} = 0.0214285714 \approx 0.021429$$

$$\delta_{c-d} = \frac{\Delta_c + \Delta_d}{|c-d|} = \frac{0.08 + 1}{|145 - 28.6|} = \frac{1.08}{116.4} = 0.00927835051 \approx 0.009278$$

$$\delta_y = \delta_{a+b} + \delta_m + \frac{1}{2} * \delta_{c-d} = 0.000796 + 0.021429 + 0.5 * 0.009278$$

$$= 0.026864$$

Bu degani nisbiy xatolik $\delta_y = 0.026864 * 100\% = 2.6864\%$.

$$y = \frac{(a+b) * m}{\sqrt{c-d}} = \frac{(23.16 + 8.23) * 0.28}{\sqrt{145 - 28.6}} = \frac{31.39 * 0.28}{\sqrt{116.4}} = \frac{8.7892}{10.788881}$$

$$= 0.8146535307971 \approx 0.814634$$

$$\Delta_y = y * \delta_y = 0.814634 * 0.026864 = 0.02188432778 \approx 0.021884.$$

Javob: Absolyut xatolik $\Delta_y = 0.021884$ va nisbiy xatolik $\delta_y = 2.6864\%$.

2-usul.

$$y_{max} = \frac{(23.18 + 8.235) * 0.286}{\sqrt{144.92 - 29.6}} = \frac{8.98469}{\sqrt{115.32}} = 0.8366634177557 \approx 0.836663.$$

$$y_{min} = \frac{(23.14 + 8.225) * 0.274}{\sqrt{145.08 - 27.6}} = \frac{8.59401}{\sqrt{117.48}} = 0.7928917257298 \approx 0.792892.$$

$$y_{ABS} = \frac{y_{max} - y_{min}}{2} = \frac{0.836663 - 0.792892}{2} = \frac{0.043771}{2} = 0.0218855.$$

$$y_{nis} = \frac{y_{ABS}}{\frac{y_{max} + y_{min}}{2}} = \frac{0.0218855}{\frac{0.836663 + 0.792892}{2}} = \frac{0.0218855}{0.8147775}$$

$$= 0.026860707371 \approx 0.026861.$$

Bu degani nisbiy xatolik $y_{nis} = 0.026861 * 100\% = 2.6861\%$

Javob: Absolyut xatolik $y_{ABS} = 0.0218855$ va nisbiy xatolik $y_{nis} = 2.6861\%$

Xulosa qilib aytganda, ixtiyoriy misol yoki masalani sonli yechishda biz aniq yechimga ega bo'lmisdan, balki u yoki bu darajadagi aniqlikda yechamiz ekan. Ushbu maqolada sonlar yig'indisi, ayirmasi, ko'paytmasi va bo'linmasining absolyut va nisbiy xatoliklarini topishni o'rganib chiqdik.

FOYDALANILGAN ADABIYOTLAR.

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