INTEGRAL TENGLAMALAR VA ULARNI YECHISH USULLARI

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Annotasiya: Integral tenglamaning ta'rifi, ko'rinishi va uning turlari beriladi, shu bilan birga integral tenglamani yechish usullari ko'rib chiqiladi.

Kalit so'zlar: Integral tenglama, Fredgolm 1-tur tenglamasi, Fredgolm 2-tur tenglamasi, Fredgolmning 3-tur tenglamasi, Volterraning 1-tur tenglamasi, Volterraning 2-tur tenglamasi, aynigan yadro, yechish usullari.

Ta'rif: Agar tenglamadagi noma'lum funksiya shu funksiyaning argumenti bo'yicha olinadigan integral ishorasi ostida bo'lsa, bunday tenglama integral tenglama deb ataladi.

Ta'rif: Ushbu integral tenglama Fredgolm1ning 1-tur tenglamasi deyiladi:

$$\lambda \int_a^b K(x,y)\varphi(y)dy = f(x) \qquad (1)$$

Bunda φ(x)– noma'lum funksiya, f(x) –ozod had va K(x,y) tenglamaning yadrosi – ma'lum funksiyalar, integrallash chegaralari a va b berilgan haqiqiy o'zgarmas sonlar.

Ta'rif: Ushbu integral tenglama Fredgolmning 2-tur tenglamasi deyiladi:

$$\lambda \int_a^b K(x,y)\varphi(y)dy + f(x) = \varphi(x) \qquad (2)$$

Bunda φ(x)– noma'lum funksiya integral ishorasidan tashqarida ham ishtirok etmoqda. (1) va (2) dagi λ tenglamaning parametri deyiladi. Bu tenglamalardagi f(x) funksiya I($a \le x \le b$) kesmada, K(x,y) yadro esa R($a \le x \le b$, $a \le y \le b$) yopiq sohada berilgan deb hisoblanadi.

Ta'rif: Agar I kesmada f (x) ≡ 0 bo'lsa, (2) tenglama quyidagi ko'rinishga keladi:

$$\lambda \int_a^b K(x,y)\varphi(y)dy = \varphi(x) \qquad (3)$$

Bunday tenglama bir jinsli integral tenglama deyiladi

Ta'rif: Ushbu integral tenglama Fredgolmning 3-tur tenglamasi deyiladi:

$$\lambda \int_a^b K(x,y)\varphi(y)dy + f(x) = \psi(x)\varphi(x) \qquad (4)$$

Agar I kesmada

a) ψ (x) ≡ 0 bo'lsa, undan (1) tenglama;

b) ψ (x) ≡ 1 bo'lsa, undan (2) tenglama kelib chiqadi.

Integral tenglamada ishtirok etadigan noma'lum funksiya ko'p argumentli, jumladan ikki argumentli bo'lishi ham mumkin.

Masalan:

$$\lambda \int_a^b \int_c^d K(x,y,t_1,t_2)\varphi(t_1,t_2)d\,t_1 dt_2 + f(x,y) = \varphi(x,y) \qquad (5)$$

bu yerda f(x,y) funksiya R(a ≤ x ≤ b , c ≤ y ≤ d) sohada, K(x,y, t_1, t_2) yadro esa P(a ≤ x ≤ b , c ≤ y ≤ d , a ≤ t_1≤ b , c ≤ t_2≤ d)sohada berilgan deb hisoblanadi; a,b,c,d va λ lar berilgan o'zgarmas haqiqiy sonlardir.

Ta'rif: Ushbu integral tenglama Volterraning 1-tur tenglamasi deyiladi:

$$\lambda \int_a^b K(x,y)\varphi(y)dy = \varphi(x) \qquad (6)$$

Bunda φ(x)– noma'lum funksiya, f(x) –ozod had I(a ≤ x ≤ b) kesmada, va K(x,y) tenglamaning yadrosi – R(a ≤ x ≤ b , a ≤ y ≤ x) yopiq sohada berilgan deb hisoblanadi..

Ta'rif: Ushbu integral tenglama Volterraning 2-tur tenglamasi deyiladi:

$$\lambda \int_a^x K(x,y)\varphi(y)dy + f(x) = \varphi(x) \qquad (7)$$

Bunda φ(x)– noma'lum funksiya integral ishorasidan tashqarida ham ishtirok etmoqda. (1) va (2) dagi λ tenglamaning parametri deyiladi.

Ta'rif: Agar I kesmada f (x) ≡ 0 bo'lsa, (2) tenglama quyidagi ko'rinishga keladi:

$$\lambda \int_a^x K(x,y)\varphi(y)dy + f(x) = \varphi(x) \qquad (8)$$

Bunday tenglama bir jinsli integral tenglama deyiladi. Integral tenglamada ishtirok etadigan noma'lum funksiya ko'p argumentli, jumladan ikki argumentli bo'lishi ham mumkin.

Masalan: $\lambda \int_a^x \int_c^y K(x,y,t_1,t_2)\varphi(t_1,t_2)d\,t_1 dt_2 + f(x,y) = \varphi(x,y)$ (9)

bu yerda f(x,y) funksiya R(a ≤ x ≤ b , c ≤ y ≤ d) sohada, K(x,y, t_1, t_2) yadro esa P(a ≤ x ≤ b , c ≤ y ≤ d , a ≤ t_1, ≤ x , c ≤ t_2≤ y) sohada berilgan deb hisoblanadi.

Ta'rif: Fredgolmning 2-tur tenglamasi berilgan bo'lsin:

$$\lambda \int_a^b K(x,y)\varphi(y)dy + f(x) = \varphi(x) \qquad (2)$$

Agar bu tenglamada ishtirok etayotgan yadroni ushbu:

$$K(x,y) = a_1(x)b_1(y) + a_2(x)b_2(y) + \ldots + a_n(x)b_n(y) \qquad (10)$$

ko'rinishida yozish mumkin bo'lsa, bunday yadro aynigan yadro deyiladi.

Integral tenglamalarni yechishning quyidagi usullari mavjud:

1. Differensial tenglamalarga keltirib yechish;
2. Aynigan yadroli integral tenglamalarni chiziqli algebraik tenglamalar sistemasiga keltirib yechish;
3. Aynigan yadroli integral tenglamalarni koeffisiyentlarni tenglash usuli bilan yechish;
4. Ketma-ket yaqinlashish usuli bilan yechish;
5. Rezolventa usuli bilan yechish. Shu usullardan ba'zilarini misollarda ko'rib chiqamiz.

Bu usullar yordamida bir necha misollarni ishlash yo'llarini ko'rib chiqaylik.

1. Ushbu tenglamani yeching.

$$\lambda \int_0^\pi K(x,y)\varphi(y)dy + f(x) = \varphi(x)$$

Bu yerda

$$K(x,y) = \sin(2x+y)\,, \quad f(x) = \pi - 2x$$

$f(x)$ funksiya, $K(x,y)$ yadro berilgan ularni Fredgolmning 2-tur tenglamasiga olib borib qo'yamiz

$$\lambda\int_0^\pi \sin(2x+y)\,\varphi(y)dy+\pi-2x=\varphi(x)$$

$$\lambda\int_0^\pi (\sin 2xcosy+sinycos2x)\,\varphi(y)dy+\pi-2x=\varphi(x)$$

Chap tomondagi qavslarni ochib ikkala intengralni ham qisqacha Q_1 va Q_2 orqali belgilaymiz:

$$Q_1=\int_0^\pi cosy\varphi(y)dy$$

$$Q_2=\int_0^\pi siny\varphi(y)dy$$

Shunda quyidagicha tenglamaga ega bo'lamiz

$$\varphi(x)=\lambda sin2xQ_1+\ \lambda cos2xQ_2+\ \pi-2x$$

$\varphi(x)$ tenglamadan $\varphi(y)$ tenglamani hosil qilib olamiz

$$\varphi(y)=\lambda sin2yQ_1+\ \lambda cos2yQ_2+\ \pi-2y$$

Hosil bo'lgan tenglamamizni Q_1 va Q_2 larga olib borib, Q_1 va Q_2 larni qiymatini topamiz

$$Q_1=\int_0^\pi cosy\varphi(y)dy=$$

$$=\int_0^\pi cosy(Q_1\lambda sin2y+\ \lambda Q_2cos2y+\ \pi-2y)dy$$

$$=2\lambda\int_0^\pi Q_1sinycos^2ydy+\ \lambda Q_2\int_0^\pi cos^3ydy-\lambda Q_2\int_0^\pi sin^2y\,cosydy$$

$$+\int_0^\pi cosy(\pi-2y)dy=\lambda Q_1\frac{4}{3}-4$$

$$Q_1=\lambda Q_1\frac{4}{3}-4$$

$$Q_1\left(1-\lambda\frac{4}{3}\right)=4$$

$$Q_1=\frac{12}{3-4\lambda}$$

Q_1 topildi endi esa Q_2 ni hisoblaylik

$$Q_2=\int_0^{\pi} siny\varphi(y)dy=$$

$$=\int_0^{\pi} siny(\lambda sin2yQ_1+\ \lambda cos2yQ_2+\ \pi-2y)dy$$

$$=\int_0^{\pi}(2\lambda Q_1sin^2ycosy+\ \lambda Q_2cos^2ysiny-\lambda Q_2sin^3y+\ \pi siny$$

$$-2ysiny)dy=$$

$$=(2\lambda Q_1\frac{sin^3y}{3}+\lambda Q_2\frac{cos^3y}{3}+\lambda Q_2\frac{3siny-sin3y}{4}-\pi cosy$$

$$+2ycosy-2siny)|_0^{\pi}=\lambda Q_2\frac{2}{3}+2\pi-2\pi=\lambda Q_2\frac{2}{3}$$

$$Q_2=0$$

Demak, berilgan tenglamamizni umumiy yechimi quyidagicha

$$\varphi(x)=\lambda sin2x(\frac{12}{3-4\lambda})+\ \pi-2x$$

Ko’rinishda bo’lar ekan.

2. Ushbu tenglamani yechaylik.

$$u(x,y)=\frac{xy}{2}-\frac{1}{3}+\int_0^1\int_0^1(xy+t_1t_2)u(t_1,t_2)dt_1dt_2$$

Aynigan yadroli ushbu integral tenglamani koeffisiyentlarni tenglash usuli bilan yechamiz.

O‘ng tomondagi qavslarni ochib birinchi intengralni Q_1 va ikkinchi integralni esa Q_2 orqali belgilaymiz:

$$u(x,y)=\frac{xy}{2}-\frac{1}{3}+xy\int_0^1\int_0^1 u(t_1,t_2)dt_1dt_2+\int_0^1\int_0^1 t_1t_2u(t_1,t_2)dt_1dt_2=\frac{xy}{2}-$$

$$-\frac{1}{3}+xyQ_1+Q_2$$

$$\left(Q_1+\frac{1}{2}\right)xy+\left(Q_2-\frac{1}{3}\right)=\alpha xy+\beta$$

u ning mana shu ifodasini berilgan integral tenglamaga qo‘yamiz:

$$\alpha xy+\beta=\frac{xy}{2}-\frac{1}{3}+\int_0^1\int_0^1(xy+t_1t_2)(\alpha t_1t_2+\beta)dt_1dt_2$$

Bu yerdagi integrallar hisoblab chiqilsa, quyidagi ayniyat

$$\alpha xy+\beta=\left(\frac{1}{4}\alpha+\beta+\frac{1}{2}\right)xy+\left(\frac{1}{9}\alpha+\frac{1}{4}\beta+\frac{1}{3}\right)$$

hosil bo‘ladi. Uning ikki tomonidagi xy ning koeffisentlarini o‘zaro hamda ozod hadlarni o‘zaro tenglash natijasida quyidagi tenglamalar

$$\alpha=\left(\frac{1}{4}\alpha+\beta+\frac{1}{2}\right),\qquad \beta=\left(\frac{1}{9}\alpha+\frac{1}{4}\beta+\frac{1}{3}\right)$$

yoki

$$\begin{cases}\frac{3}{4}\alpha-\beta=\frac{1}{2}\\ \frac{1}{9}\alpha-\frac{3}{4}\beta=\frac{1}{3}\end{cases}$$

chiziqli algebraik tenglamalar sistemasi hosil bo‘ladi. Bu sistemaning yechimi

$$\alpha=\frac{6}{65}\qquad \beta=-\frac{28}{65}$$

Demak, integral tenglamaning yechimi

$$u(x,y)=\alpha xy+\beta=\frac{6}{65}xy-\frac{28}{65}$$

bo‘ladi.

FOYDALANILGAN ADABIYOTLAR.

1. SH.Merajova “Matematik fizika tenglamalari fanidan masalalar to’plami” uslubiy ko’rsatma.

2. M.S.Salohiddinov, B.I.Islomov “Matematik fizika tenglamalari fanidan masalalar to’plami”.

3. O.Zikirov “matematik fizika tenglamalari”.

4. Yuldashev T. K., Kholmanova K. Y. “Nonlinear second order Fredholm integro-differential equation”.

5. Yuldashev T. K., Kholmanova K. Y “NONLINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATI WITH DEGENERATE KERNEL AND NONLINEAR MAXIMA”

6. Xolmanova K.Y “Maksimumli chiziqsiz differensial tenglamalar haqida”.

7. Xolmanova K.Y. “Maksimum belgisi ostida berilgan funksiyani hosilasini o’z ichiga oluvchi integro-differensial tenglamalar sistemasi uchun boshlang’ich masala”.

8. Xolmanova K.Y “Integral belgisi ostida funksional parametrni o’z ichiga olgan integro-defferensial tenglamalar sistemasi uchun berilgan kesmada boshlang’ich masala”.

9. Юлдашев Т.К. Краевая задача для нелинейных интегро-дифференцыальных уравнений параболо-гиперболического типа второго порядка с максимумами / Там же. –С. 24-29.

10. Юлдашев Т.К. Начальная задача с условием склеивания для нелинейных дифференцыальных со смешанными максимумами / Туранск. Матем. Собр.-Ош, 1995.-Вып. 1. –С.5-13.