THERMODYNAMIC POTENTIAL OF THE BOSE GAS

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Annotatsiya. Ushbu maqolada Boze kondensatsiya yaqinida ximik potensialning temperaturaga bog'lanish masalasi nazariy jihatdan tahlil etilgan. Boze kondensatsiyadan pastki sathda ushbu bog'lansh mikroskopik tabiatga ega. Boze kondensatsiyadan yuqori sathlarda esa ximik potentsial kondensat zarralari soniga teskari proportsional ekan.

Kalit so'zlar: Boze kondensatsiya, tenglama, zarrachalar, potentsial.

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Аннотация.Исследована зависимость химического потенциала от температуры в окрестности и ниже точки перехода Бозе конденсации. Следует отметить, что ниже точки бозе-конденсации химический потенцал имеет микроскопический порядок. Ниже точки бозе-конденсации химический потенциал обратно пропорционален числу частиц конденсата.

Ключевые слов. Бозе конденсация, уравнение,частицы, потенциал

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Abstract. In this paper we investigated and the dependence of the chemical potential on temperature in the vicinity of and below the transition Bose condensation. It should be noted that below the Bose condensation chemical potential has microscopic order. It was shown that below bose condensation point chemical potential is inversely proportional to the number of condensate particles.

Keywords: Bose condensation, transition , equation, Bose particles

INTRODUCTION

Distribution function of Bose particles is easy to obtain, considered the thermodynamic potential of the system

$$\Omega = -T \ln Z, \tag{1}$$

where the partition function Z in the grand canonical ensemble for a system of nevz and moderating particles has the form

$$Z = \sum_N e^{\frac{\mu N}{T}} \sum_n e^{-\frac{E_{nN}}{T}} = \sum_{n_k} \left(e^{\beta(\mu-\varepsilon_k)}\right)^{n_k}, \tag{2}$$

where $\varepsilon_k n_k$ the energy of n_k particles in state k and $\mu < 0$ is the chemical potential [3-5]. Now you can find the average number of particles

$$\langle n_k \rangle = -\frac{\partial \Omega_k}{\partial \mu}, \qquad n_k(T) = \langle n_k \rangle = \langle a_k^+ a_k \rangle = \frac{1}{\exp\left[\frac{\varepsilon_k - \mu}{T}\right] - 1} \tag{2}$$

The expression (2) is the one-particle Bose - Einstein distribution function. The energy of one particle, is

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}.$$

The total number of particles in gas we obtain, summing up (2)

$$E = \sum_k \varepsilon_k \frac{1}{\exp\left[\frac{\varepsilon_k - \mu}{T}\right] - 1} = \frac{V m^{3/2}}{\sqrt{2}\pi^2 \hbar^3} \int_0^\infty d\varepsilon \frac{\varepsilon\sqrt{\varepsilon}}{\exp\left[\frac{\varepsilon - \mu}{T}\right] - 1} \tag{3}$$

At low temperatures, the properties of a Bose gas are fundamentally different from those of a classical system in that the ground state of the system has energy $E = 0$ (i.e., all particles are condensed into state $\varepsilon_k = 0$). According to the normalization equation (3), at lower temp and tours chemical potential increases, the remaining negative, and up to $\mu = 0$ value at a temperature T_0 that satisfies the relation

$$T_0 = \frac{2\pi}{\zeta\left(\frac{3}{2}\right)^{2/3}} \frac{\hbar^2}{m} \left(\frac{N}{V}\right)^{2/3} = 3.31 \frac{\hbar^2}{m} \left(\frac{N}{V}\right)^{2/3} \tag{4}$$

Here it is taken into account that $\int_0^\infty dz \frac{\sqrt{z}}{e^z - 1} = \Gamma(z)\zeta(z)$, where $\zeta(z)$ - the Riemann zeta function $\zeta\left(\frac{3}{2}\right) = 2.162,\ \Gamma(z)$, $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}/2.$ the gamma function From (4) we obtain the temperature (as will be seen below, the condensation temperature)

At lower temperatures $T<T_0$, normalization equation (4) has no solutions $\mu<0$, although they should exist for Bose statistics. This associated with a fact that in this case can not move formally from summation to integration in (3). It is necessary to take into account the term carefully $\varepsilon_k = 0$, and due to the presence of the multiplier $\sqrt{\varepsilon}$, it drops out of the sum.

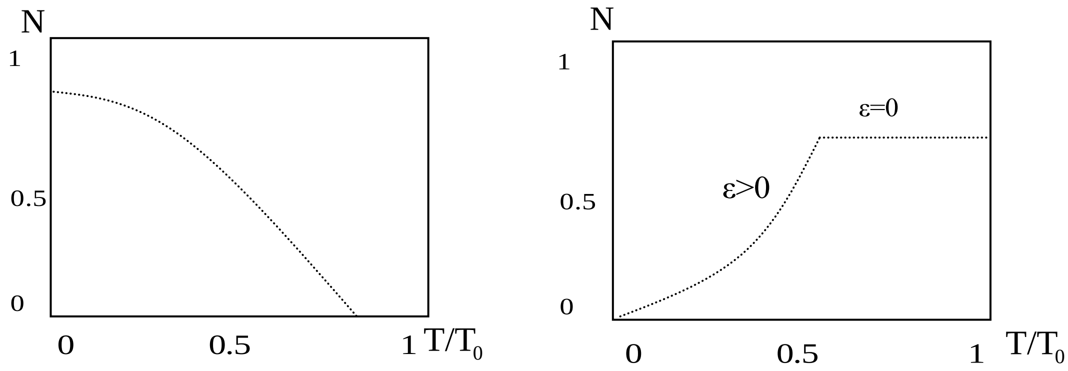

Fig. 1. The temperature dependence of the number of particles

However, it is precisely this that is important at low temperatures, since all particles condense into the state $\varepsilon_k = 0$. Formally, from (3) may be of a mark that during the transition to the limit $\mu \to 0$ this term diverges. To solve this problem in two ways: firstly, letting μ not to zero but to a small value and, secondly, by calculating first number of particles in $\varepsilon = 0$ (for $T < T_0$), since this value is determined by (5) $\mu = 0$ is limiting, of course

$$N_{\varepsilon>0} = \frac{V(mT)^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty dz \frac{\sqrt{z}}{e^z - 1} = N\left(\frac{T}{T_0}\right)^{3/2} \tag{5}$$

(здесь использовано определение T_0 (7)). Остальные сконденсированные в состояние $\varepsilon = 0$ частицы определяются из нормировки(here the definition of T_0 (3) is used). The remaining condensed state $\varepsilon = 0$ in the particles was determined by the normalization condition

$$N_{\varepsilon=0} = N_0 = N\left(1 - \left(\frac{T}{T_0}\right)^{3/2}\right). \tag{6}$$

Thus, at a temperature $T=T_0$, the condensation of Bose particles into the lowest energy state $\varepsilon = 0$ begins, and the number of condensed particles N_0 is determined by the power-law dependence (6). Hence, in particular, that with decreasing the system temperature from the critical value concentration of particles having a zero pulse. The chemical potential of a Bose gas obeys the equation

$$\frac{(2\pi\hbar)^3}{4\pi V(mT)^{3/2}}\frac{N-N_0}{2s+1} = \int_0^\infty p^2 dp \sum_{n=1}^\infty \exp\left[-\left(\frac{p^2}{2}-\frac{\mu}{T}\right)\right] = \sqrt{\frac{\pi}{2}}\sum_{n=1}^\infty n^{-3/2}\exp\left(\frac{n\mu}{T}\right) \tag{7}$$

Above the Bose condensation $\mu \le 0$ point, the chemical potential is shown in Fig. 2. The transition to the Boltzmann case is carried out if μ □ T .

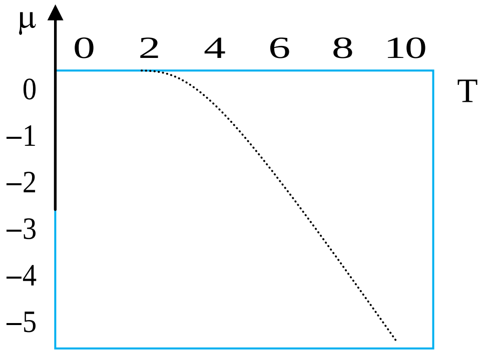

Fig. 2. The temperature dependence of the chemical potential

RESULTS AND DISCUSSIONS

We derive an analytic expression for the chemical potential for specified limiting case. For this, we present the formula

$$\frac{1}{2S+1}\left(\frac{T_0}{T}\right)^{3/2} = \exp\left(\frac{\mu}{T}\right) + \frac{1}{2\sqrt{2}}\exp\left(\frac{2\mu}{T}\right) \tag{11}$$

to the next simple form

$$T_0 = \frac{2\pi\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3} = T_c\left[\frac{1}{\varsigma\left(\frac{3}{2}\right)(2S+1)}\right]^{2/3} \tag{12}$$

Where we can easily find the expression for the chemical potential, as follows

$$\mu = T \ln \frac{1}{2S+1}\left(\frac{T_0}{T}\right)^{3/2} - \frac{T}{\sqrt{2}}\frac{1}{2S+1}\left(\frac{T_0}{T}\right)^{3/2} \qquad (13)$$

Now we find the temperature dependence of the chemical potential and critical temperature T_c. Solving equations (10) - (12) together, we find the following expression

$$\frac{1}{2S+1}\left[\left(\frac{T_0}{T}\right)^{3/2} - \left(\frac{T_0}{T_{\tilde{n}}}\right)^{3/2}\right] = \sum_{n=1}^{\infty} n^{-3/2}\left(\exp\left(\frac{\mu n}{T}\right) - 1\right) \qquad (14)$$

After simple analytical transformations

$$-\frac{3}{2S+1}\left(\frac{T_0}{T_{\tilde{n}}}\right)^{3/2}\frac{T_0 - T_c}{T_c} = \int_1^{\infty} dn\, n^{-3/2}\left(\exp\left(\frac{\mu n}{T}\right) - 1\right) = \sqrt{-\frac{\mu}{T}}\int_0^{\infty} dn\, n^{-3/2}\left(\exp(-n) - 1\right) = -2\sqrt{-\frac{\mu\pi}{T}}$$

we find the formula for the chemical potential

$$\mu = -\frac{T_c}{\pi}\left(\frac{T - T_c}{T_c}\right)^2\left[\frac{3}{4\varsigma\left(\frac{3}{2}\right)(2S+1)^2}\right]^2$$

Thus, we have obtained an expression for the temperature-dependence of chemical potential near the critical temperature. It should be noted that below the Bose condensation point the chemical potential has microscopic order. Below Bose condensation point chemical potential is inversely proportional of the number of condensate particles

$$|\mu| = \frac{T}{N}\left(1 - \left(\frac{T}{T_c}\right)^{-3/2}\right)$$

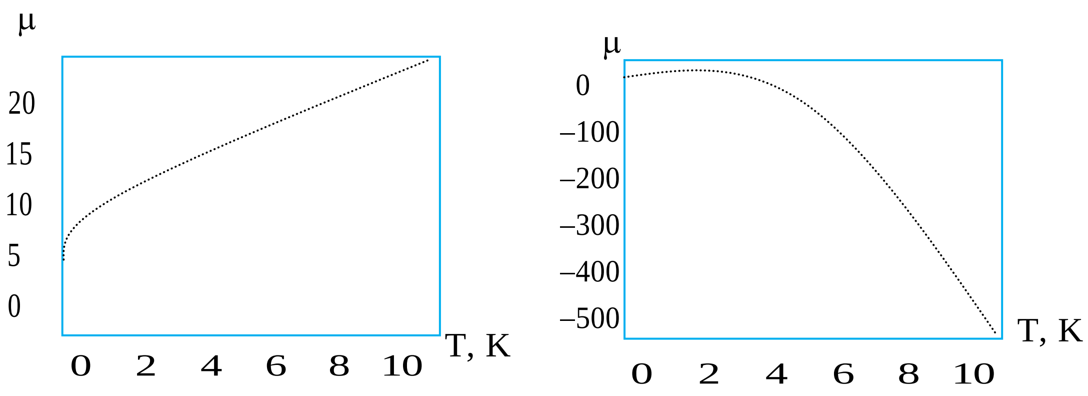

Fig. 3. The temperature dependence of the chemical potential

The behavior of the chemical potential in the vicinity of and below the point of crossing can be presented following interpolation formula

$$|\mu| = \frac{T}{N}\left(1-\left(\frac{T}{T_c}\right)^{-3/2} + \left(\frac{2\sqrt{\pi}}{\varsigma\left(\frac{3}{2}\right)}\right)^{-2/3}\right)$$

In fig. 3 is a plot of the chemical potentials on temperature in the vicinity of and below the point of transition.

CONCLUSION

In this paper we investigated and the dependence of the chemical potentials of the temperature in the vicinity of and below the transition Bose condensation. It should be noted that below the Bose condensation chemical potential a microscopic order and inversely proportional to the number of particles condensate.

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